Can the Market Multiply and Divide?

Non-Proportional Thinking in Financial Markets

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Proportional thinking in financial markets

In financial markets, rational investors should react to news about firm value in terms of proportional price changes, i.e. **returns**

Market value of the firm:

Size = number of shares × share price

Holding firm size constant, nominal price of a financial security has no real meaning

- Price can easily be changed through splits or reverse splits

But changes in the value of stocks are frequently reported in dollar units rather than or in addition to return units…

Wall Street Journal (1970s)

Tuesday's Volume, 16,056, Shares

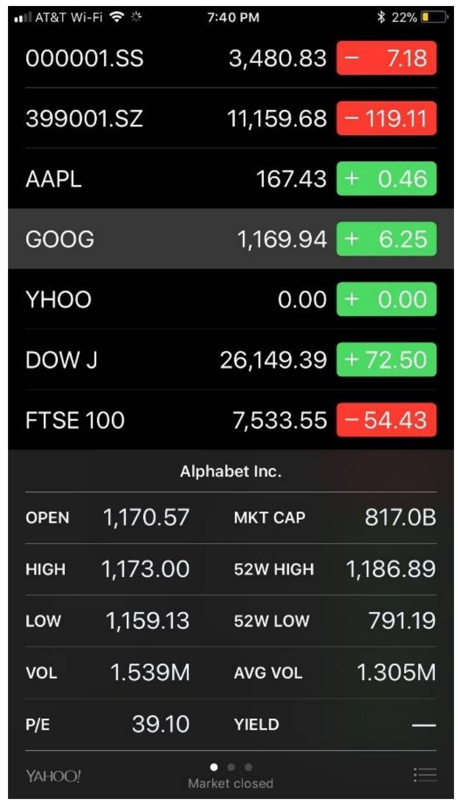
Volume since Jan. 1:	1970	1969	1968
Total sales	258,654,878	291,760,651	301,114,208

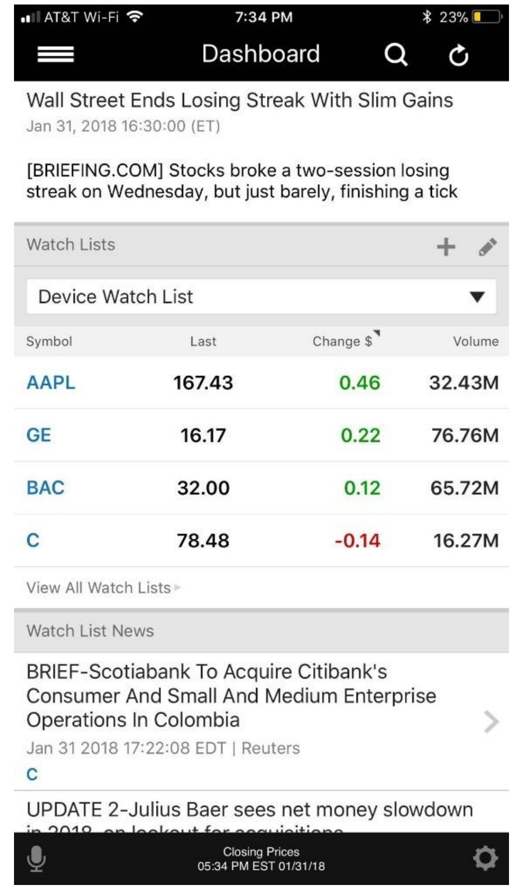
MOST ACTIVE STOCKS

	Open	High	Low	Close	Chg.	Volume
Chrysler	24¼	25⅝	24¼	25½	+1¼	335,100
Cont Data	71¼	71¼	63½	67⅛	−4½	277,500
CNA Finl	18⅝	18⅝	17	18⅛	− ¼	230,500
Texaco	25⅝	25⅞	24¾	25¾	+ ½	67,700
Telex Corp	134¼	135¾	126½	134⅝	−1¾	61,400
Am Airlin	23⅝	23⅞	21	22½	−1½	50,300
Itek Corp	79½	82	76½	79¾	−2⅛	42,500
Mid So Util	21	21	20	20	− ¾	33,100
Unvsty Cmp	56⅝	61⅞	53¼	59½	+1⅞	32,500
Teledyne	28¼	28½	20	27½	− ¾	30,700

Average closing price of most active stocks: 48.01.

Android, Apple, and Etrade apps (2010s)





CNBC (2010s)


SQUAWK BOX
THE PRIVATE EQUITY LANDSCAPE
S&P FUT (Jun) -3.25
S&P FV 2.83
S&P CLOSE 2,047.63
ap Bear 3X (TZA) 43.33 ▲ 0.39 iPath S&P VIX (VXX) 15.82 ▲ 0.24 Direxi
MGT Capital Invest (MGT) 2.82 ▲ 0.20 MGT Capital Invest (MGT) 2.81 ▲
CNBC
8:10A EASTERN

Our hypothesis

Non-proportional thinking: Investors think that news should correspond to a dollar change in price rather than a percentage change

- Motivated by experimental evidence (Svedsater et al. 2007)

Consider two otherwise identical stocks, one trading at $20/share, another at $30/share

- Investors may think the same piece of news should correspond to a $1 increase in price for both stocks

→ Overreaction to news for low-priced stocks: return reaction to news is (relatively) too big

→ Underreaction to news for high-priced stocks: return reaction to news is (relatively) too small

Volatility predictions

Measures of a stock’s volatility:

1. Total volatility: standard deviation of returns
2. Idiosyncratic volatility: standard deviation of returns in excess of market returns
3. Beta: scaled covariance between the stock’s return and the market return

Return overreaction for lower priced stocks → These stocks will have greater total volatility, idiosyncratic volatility, and absolute beta

Preview of results

A doubling in share price corresponds to 20-30% decline in volatility (total volatility, idiosyncratic volatility, and market beta)

Not driven by size—rather, the size-volatility relation flattens by 80% after controlling for price

To identify a causal effect of price, we show that volatility jumps immediately after stock splits and drops after reverse splits

Non-proportional thinking also distorts investors reactions to news that is reported in nominal rather than the appropriate proportional units: e.g. nominal earnings surprises

Not driven by tick-size limitations, volume, liquidity, or changes to a speculative investor base

Implications

A new explanation of under and overreaction to news

- Complements other behavioral explanations which focus on limited attention, biased beliefs about persistence

Offers insight into the determinants of volatility and drift

- Well-known asset pricing facts such as “small stocks have higher volatility and beta” are mostly driven by price
- Potential explanation for the “leverage effect” puzzle in which volatility is negatively related to past returns
- Long run reversals and predictability

Data

- CRSP
- CompuStat
- I/B/E/S: Quarterly earnings surprises relative to analyst forecasts
- Optionmetrics
- Thomson One: Institutional ownership data
- Ken French Data Library: size cutoffs, market variables
- Barber and Odean (2000)

Baseline regression

$$log(vol_{it}) = \beta_0 + \beta_1 log\left(price_{i,t-1}\right) + controls + \tau_t + \varepsilon_{it}$$

- Stock i in year-month t
- vol_{it}: **total volatility**, **idiosyncratic volatility** or **absolute market beta**
- $controls$ can include size (linear control or 20 size categories), sales volatility, volume, institutional ownership, market-to-book, leverage, firm FE
- Standard errors double-clustered by stock and year-month

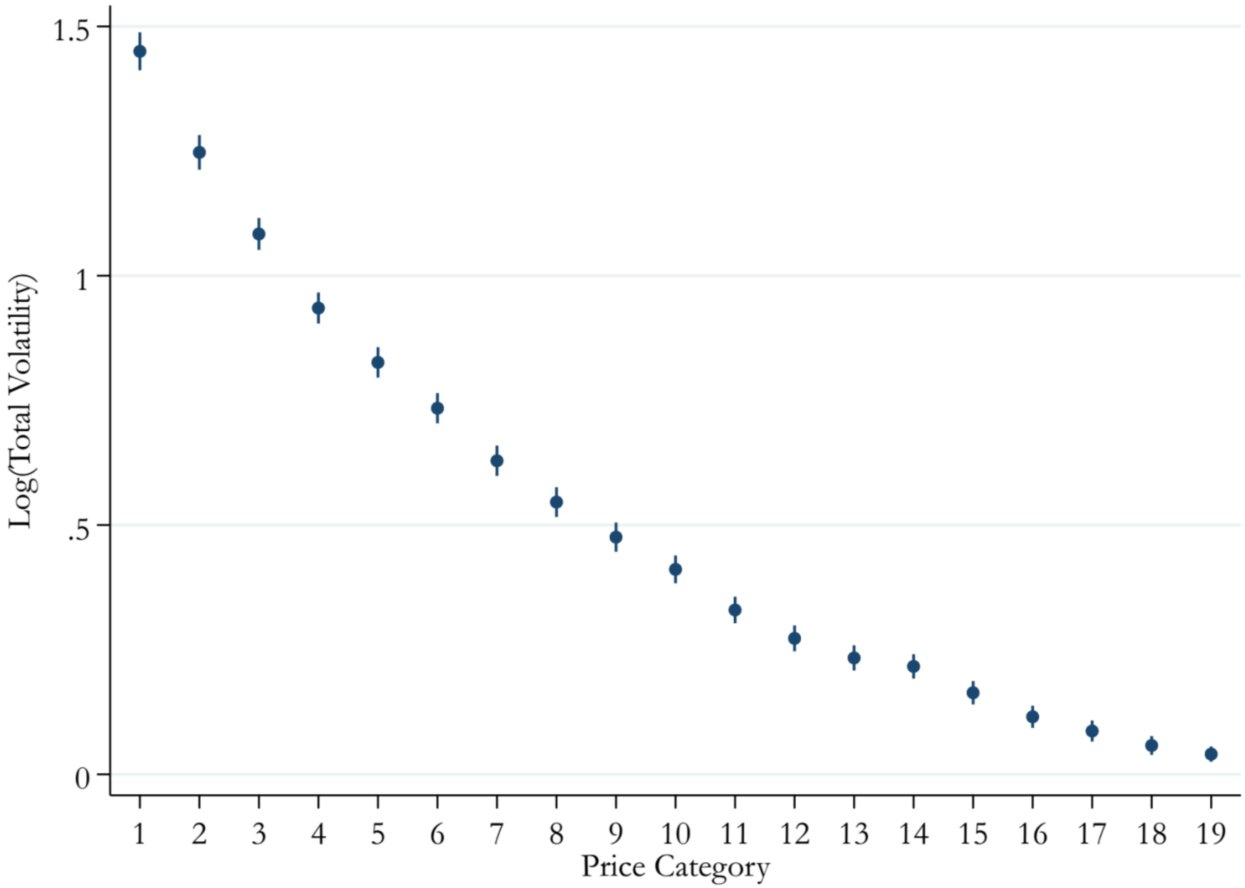
- **Non-proportional thinking predicts $\boldsymbol{\beta_1 < 0}$**

Baseline results

	Log(Total Volatility)				Log(IVol)	Log(|Beta|)
	(1)	(2)	(3)	(4)	(5)	(6)
Log(Lagged Price)	-0.326***		-0.332***	-0.339***	-0.346***	-0.323***
	(0.00339)		(0.00446)	(0.00405)	(0.00399)	(0.00459)
Log(Lagged Size)		-0.146***	0.00431			
		(0.00235)	(0.00311)			
Month FE	Yes	Yes	Yes	Yes	Yes	Yes
Size Category FE	No	No	No	Yes	Yes	Yes
R-squared	0.442	0.328	0.442	0.445	0.471	0.115
Observations	3,254,302	3,254,302	3,254,302	3,254,302	3,254,302	3,254,302

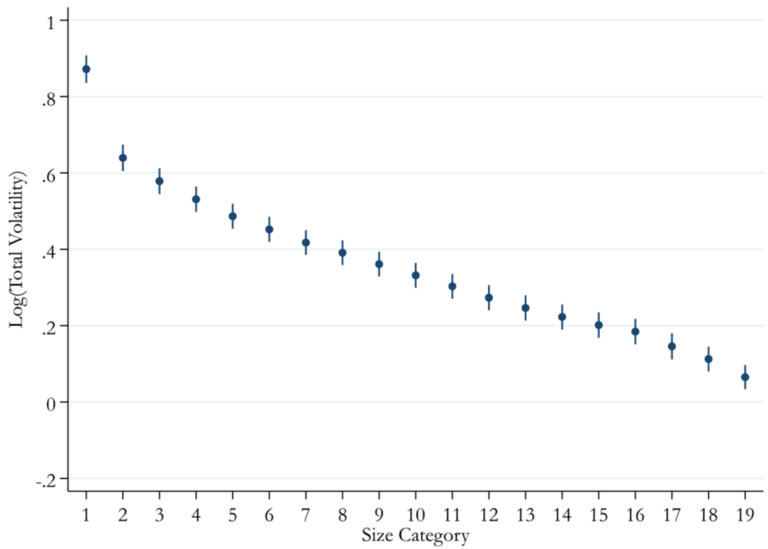
- A doubling in price corresponds to a $> 30\%$ decline in volatility
- Holds after controlling flexibly for size, yet the size-volatility relation becomes insignificant once we control for price

Non-parametric volatility-price relation

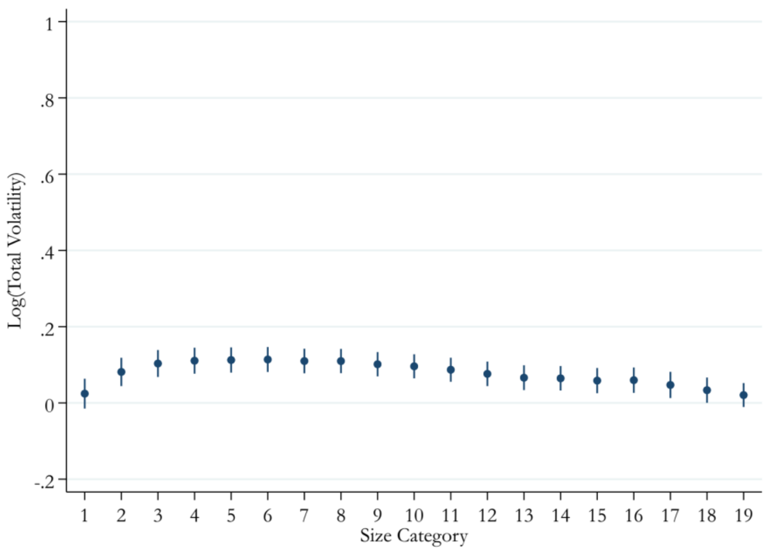


- Plots coefficients for 20 price categories (omitted category 20), controlling for 20 size categories and time FE
- Shows that negative relation is not driven only by low priced stocks that are subject to tick-size distortions

Price can explain the size-volatility relation

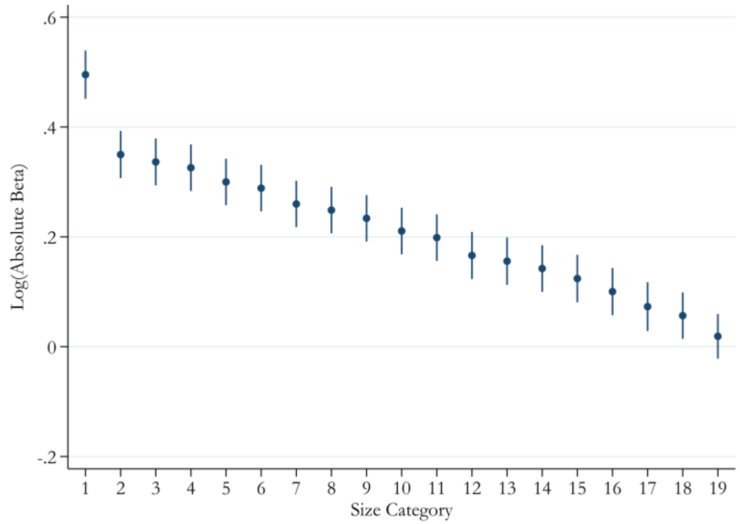


- Without controlling for $log(price_{i,t-1})$

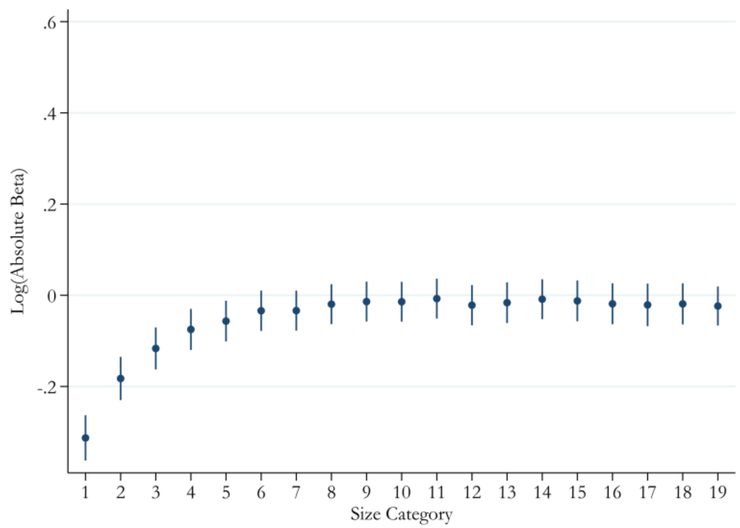


- Controlling for $log(price_{i,t-1})$
- Size-volatility relation flattens by $\sim$80%

Price can explain the size-beta relation



- Without controlling for $log(price_{i,t-1})$



- Controlling for $log(price_{i,t-1})$
- Size-beta relation flattens by $\sim 80\%$

Heterogeneity by size

	Log(Total Volatility)									
	(1) Size1	(2) Size2	(3) Size3	(4) Size4	(5) Size5	(6) Size6	(7) Size7	(8) Size8	(9) Size9	(10) Size10
Log(Lagged Price)	-0.363***	-0.386***	-0.370***	-0.364***	-0.346***	-0.325***	-0.310***	-0.298***	-0.281***	-0.270***
	(0.00562)	(0.00659)	(0.00746)	(0.00771)	(0.00800)	(0.00816)	(0.00870)	(0.00870)	(0.00949)	(0.00996)
Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.393	0.368	0.363	0.362	0.354	0.349	0.347	0.351	0.343	0.350
Observations	1,111,769	333,979	226,006	173,581	146,796	128,577	117,145	107,699	99,812	92,354

	Log(Total Volatility)									
	(1) Size11	(2) Size12	(3) Size13	(4) Size14	(5) Size15	(6) Size16	(7) Size17	(8) Size18	(9) Size19	(10) Size20
Log(Lagged Price)	-0.248***	-0.227***	-0.207***	-0.201***	-0.184***	-0.168***	-0.166***	-0.124***	-0.123***	-0.139***
	(0.00969)	(0.0103)	(0.0118)	(0.0123)	(0.0135)	(0.0139)	(0.0179)	(0.0165)	(0.0206)	(0.0215)
Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.355	0.359	0.351	0.357	0.359	0.388	0.403	0.412	0.450	0.526
Observations	85,267	81,440	78,053	75,213	72,894	70,567	67,662	65,314	62,743	57,431

Volatility-price relation isn't driven by a correlation between price and size

- But the magnitude of volatility-price relation declines with size, consistent with limits to arbitrage

Heterogeneity by decade

	Log(Total Volatility)									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	1920s	1930s	1940s	1950s	1960s	1970s	1980s	1990s	2000s	2010s
Log(Lagged Price)	-0.227***	-0.275***	-0.350***	-0.191***	-0.324***	-0.462***	-0.317***	-0.369***	-0.353***	-0.251***
	(0.0140)	(0.0108)	(0.00989)	(0.0147)	(0.0126)	(0.00862)	(0.00646)	(0.00768)	(0.00995)	(0.00554)
Month FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Size Category FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.560	0.652	0.609	0.289	0.396	0.353	0.315	0.440	0.449	0.356
Observations	22,843	82,661	97,620	118,906	209,217	452,864	624,639	751,051	586,932	307,569

Some variation over time, but the relation is not only driven by the early sample period

Robustness

The volatility-price relation isn't driven by a correlation between price and institutional ownership/size

- But, magnitude of volatility-price relation declines with size and institutional ownership, consistent with limits to arbitrage

Lower priced stocks have higher upside and downside volatility and beta, so results not driven by one tail of returns

Similar magnitudes controlling for lagged sales volatility, market-to-book, volume turnover, and leverage

Similar results after adjusting conservatively for tick-size or restricted to a subsample with zero leverage

Long run correction of initial over- and under-reaction

Non-proportional thinking predicts

- Overreaction to news for low-priced stocks and *eventual reversal*
- Underreaction to news for high-priced stocks and *eventual drift*

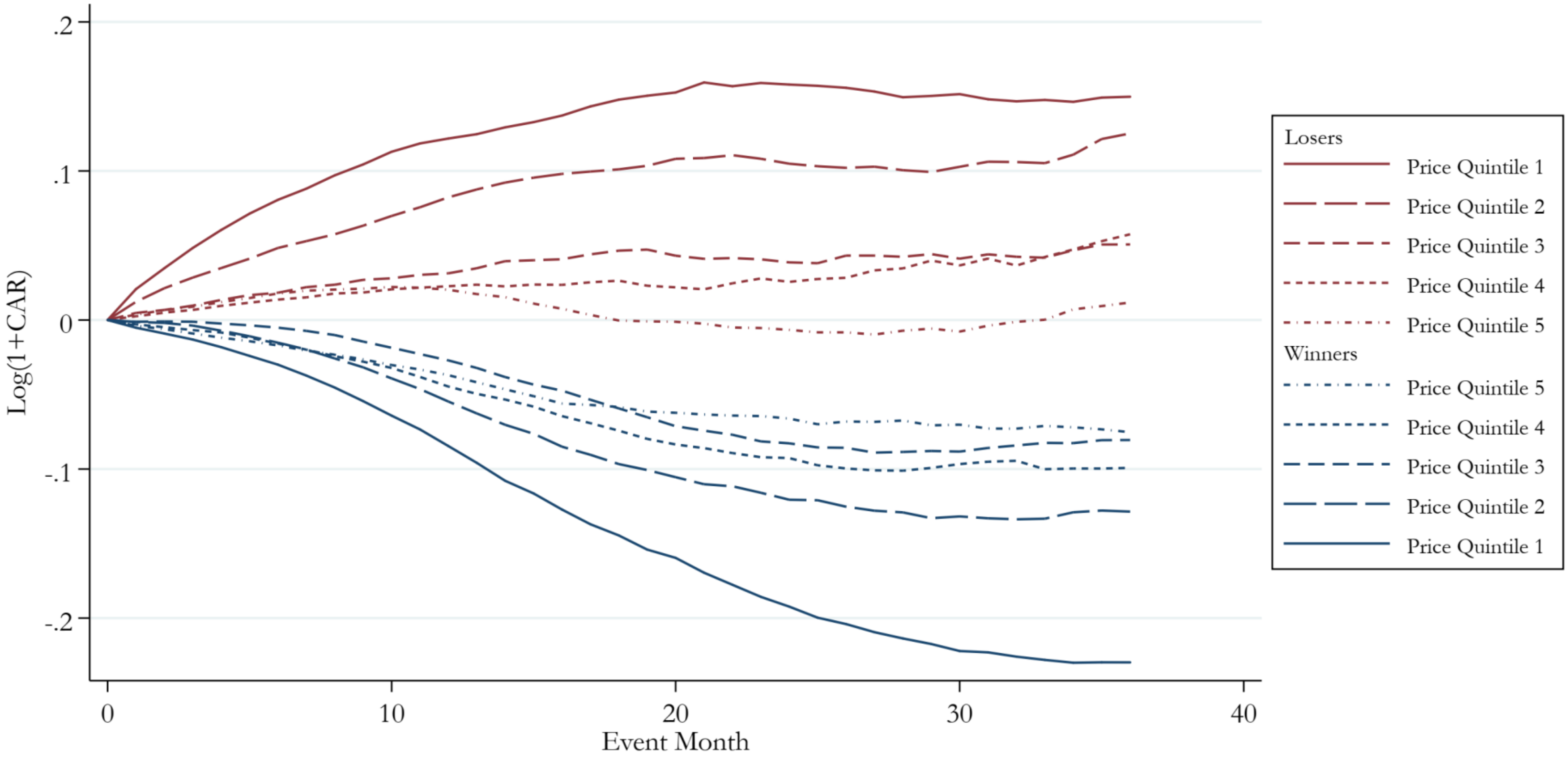
Classic evidence of long run reversals:
De Bondt and Thaler (1985) "Does the Stock Market Overreact?"

- Past winners underperform, past losers outperform

We can replicate De Bondt and Thaler (1985), and find

- The reversal is driven by low-priced stocks
- The magnitude of the reversal can be sorted by price (not size)

Long run reversals for winners and losers: by price



Future returns, double sorted by past performance and past price

	Mean Log(1 + 36-Month CAR)									
	Lagged Price Decile									
Prev 36-Month CAR Decile	Low	2	3	4	5	6	7	8	9	High
Low	0.10	0.18	0.14	0.11	0.063	0.036	0.052	0.063	0.045	-0.020
2	0.10	0.078	0.030	-0.0025	0.0018	0.035	-0.0014	0.018	0.0084	-0.035
3	0.074	0.029	0.024	0.015	0.010	0.034	0.020	-0.0073	0.020	-0.020
4	0.083	0.036	0.0011	-0.0089	0.031	0.052	0.031	0.028	0.029	-0.0019
5	0.053	0.0066	-0.0012	-0.0012	0.053	0.071	0.051	0.055	0.038	-0.026
6	0.033	0.011	0.0011	0.0053	0.066	0.089	0.079	0.055	0.043	-0.028
7	0.022	-0.015	-0.021	0.012	0.053	0.055	0.089	0.056	0.043	-0.014
8	-0.022	-0.032	-0.016	0.016	0.053	0.055	0.057	0.033	0.022	-0.024
9	-0.11	-0.076	-0.049	0.0039	0.016	0.017	-0.0012	-0.0036	-0.012	-0.046
High	-0.24	-0.22	-0.15	-0.11	-0.085	-0.076	-0.097	-0.10	-0.084	-0.061

Reversals: Price versus Size

	Log(1 + 36-Month CAR)			
	(1)	(2)	(3)	(4)
Log(1 + Prev 36-Month CAR)	-0.134***	-0.156***		
	(0.0118)	(0.0247)		
Log(1 + Prev 36-Month CAR) × Log(Lagged Price)	0.0390***	0.0360***		
	(0.00397)	(0.00481)		
Log(1 + Prev 36-Month CAR) × Log(Lagged Size)		0.00267		
		(0.00260)		
Prev 36-Month CAR Decile			-0.0406***	-0.0423***
			(0.00265)	(0.00271)
Prev 36-Month CAR Decile × Lagged Price Decile			0.00539***	0.00450***
			(0.000363)	(0.000506)
Prev 36-Month CAR Decile × Lagged Size Decile				0.00120**
				(0.000502)
Month FE	Yes	Yes	Yes	Yes
R^2	0.093	0.093	0.094	0.094
Observations	1,613,378	1,613,378	1,660,459	1,660,459

- Past returns negatively predict future returns
- The strength of this reversal varies more by price than by size

Splits as an event study

Potential concern: poor performance leads to low price and high vol

- More generally, omitted variables drive price-volatility relation

To identify a causal effect of price, we conduct a regression discontinuity around stock splits

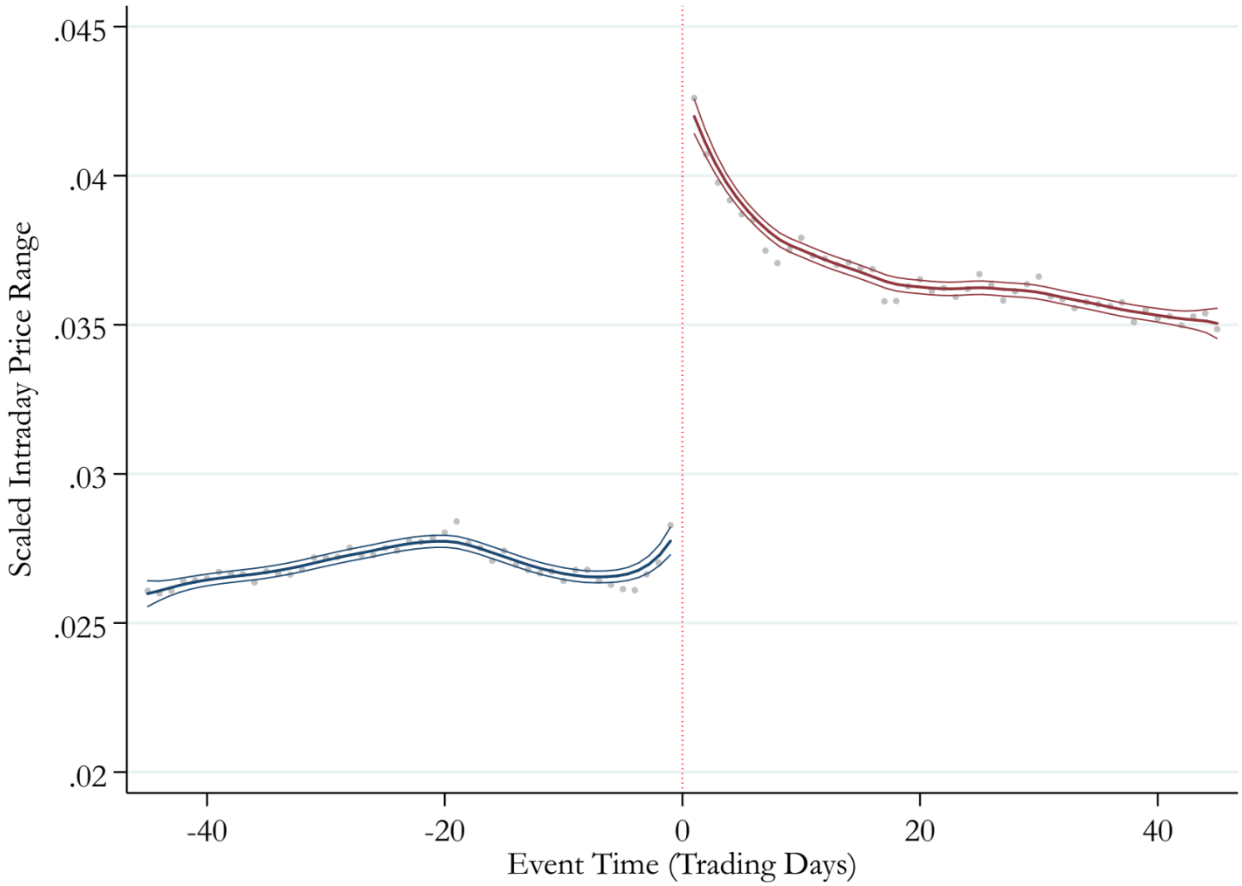
- Following a standard 2-for-1 stock split, the price falls by half

Splits are not random (they tend to follow good performance), but splits are pre-scheduled and fundamentals are unlikely to change exactly on the split date

- We verify this in the data

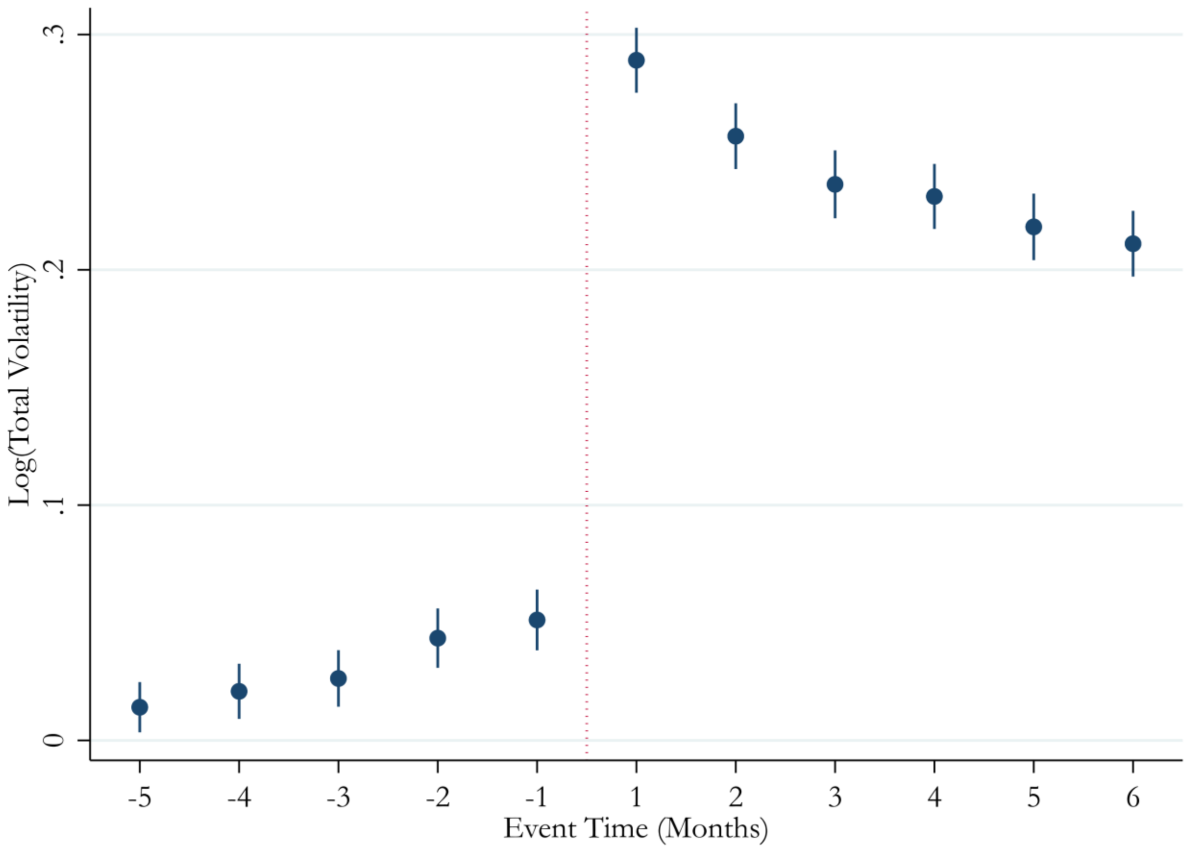
Expect the opposite patterns for reverse splits

Regression discontinuity around stock splits



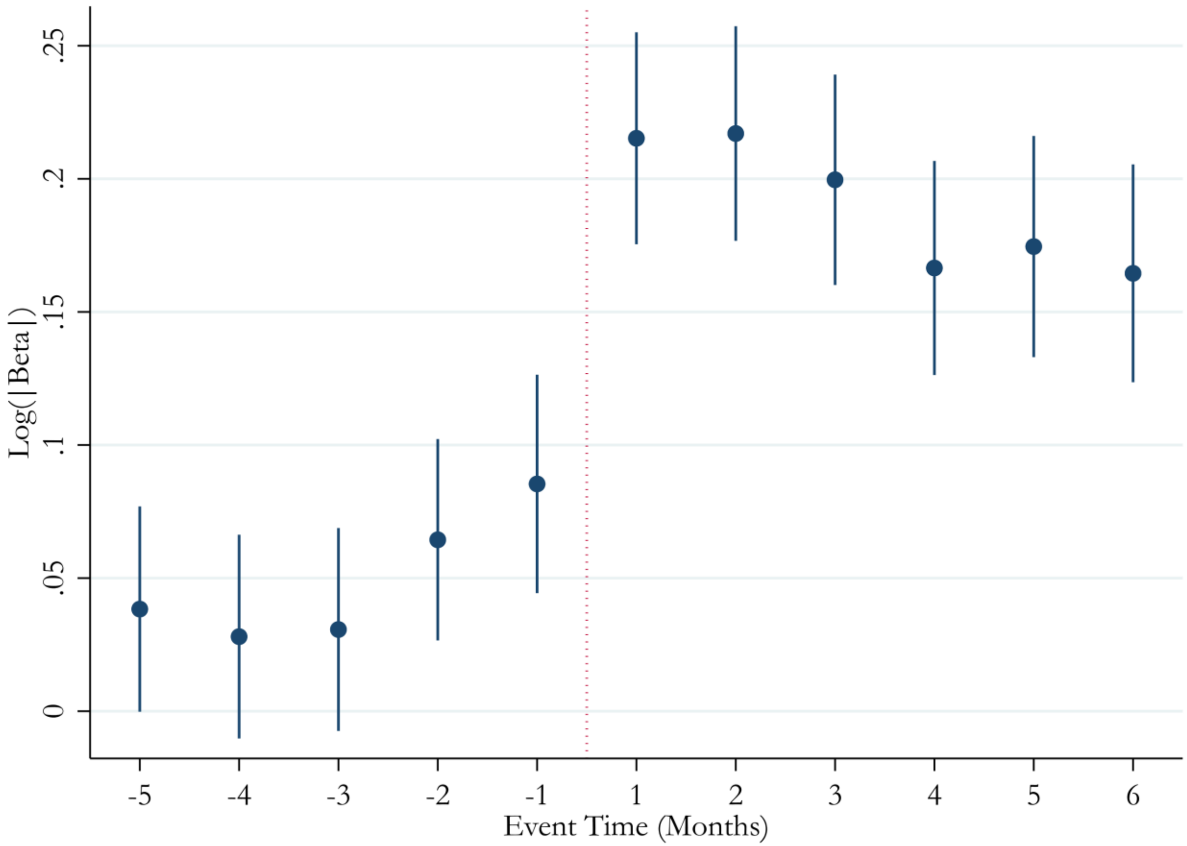
- Proxy for daily volatility using scaled intraday price range
- > 35% persistent increase in intraday price range after splits

Splits (total volatility)

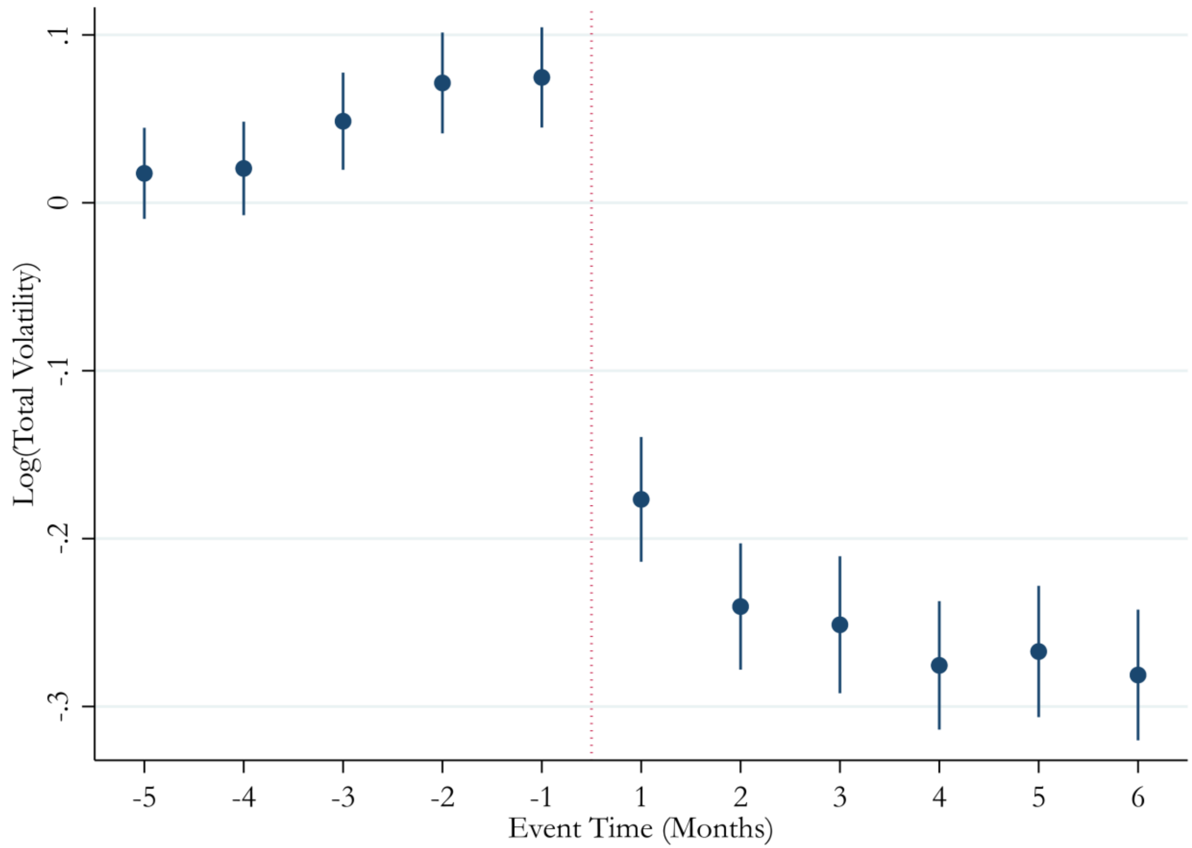


Regression coefficients on event months ($t = -6$ is omitted), controlling for stock and calendar year-month FE

Splits (absolute beta)



Reverse splits



Following reverse splits when price jumps, volatility drops

Remaining alternative explanations

Low prices may attract speculative investors who push up volatility (Brandt et al. 2009; Dhar et al. 2004)

- Unlikely that investor base changes in a single day after split
- Not obvious that speculative investors would overreact to news, leading to higher betas and subsequent reversals

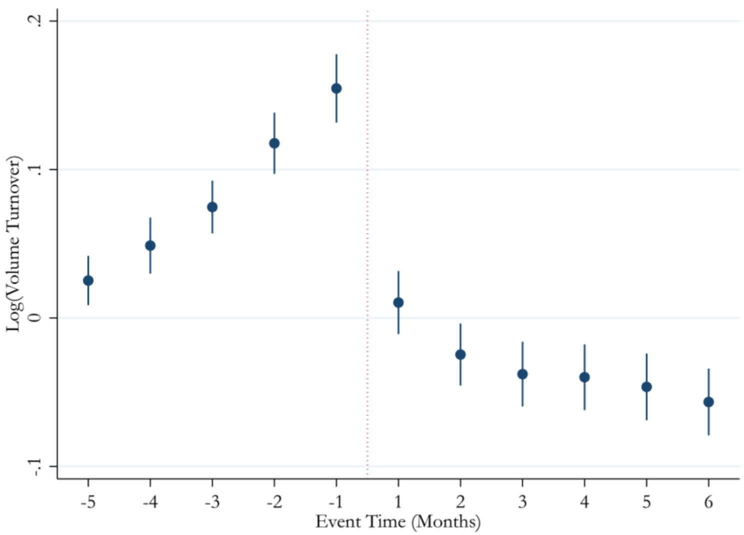
Firms may announce splits when they expect changes in firm strategy/performance, which could affect volatility

- However, splits are usually announced one month ahead

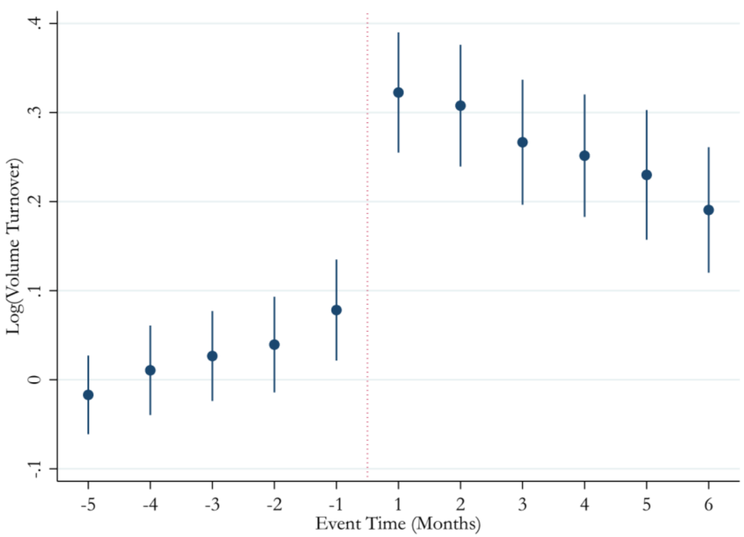
We can also examine these stories in more detail…

Volume after splits

(a) **Positive Stock Splits**



(b) **Reverse Stock Splits**



- Holding a stock's market cap constant, increased speculation should lead to higher volume
- Instead, volume drops after splits, consistent with some investors trading fixed numbers of shares

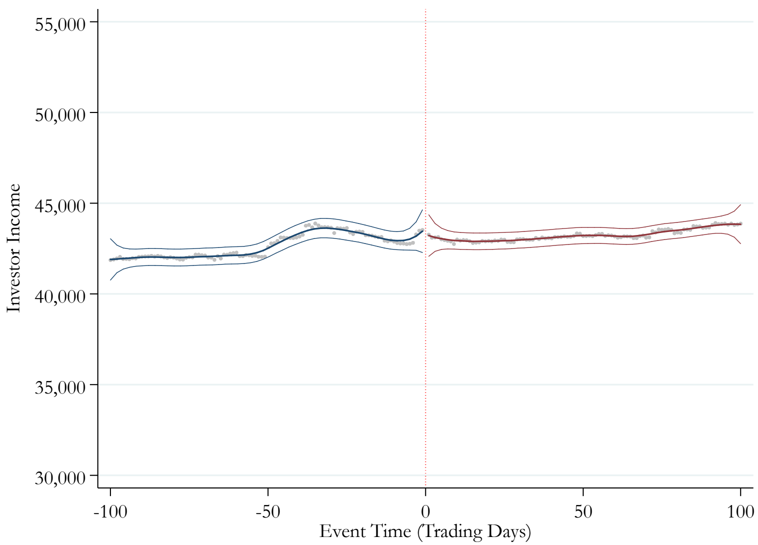
- Volume increases following reverse splits, consistent with some investors trading fixed numbers of shares

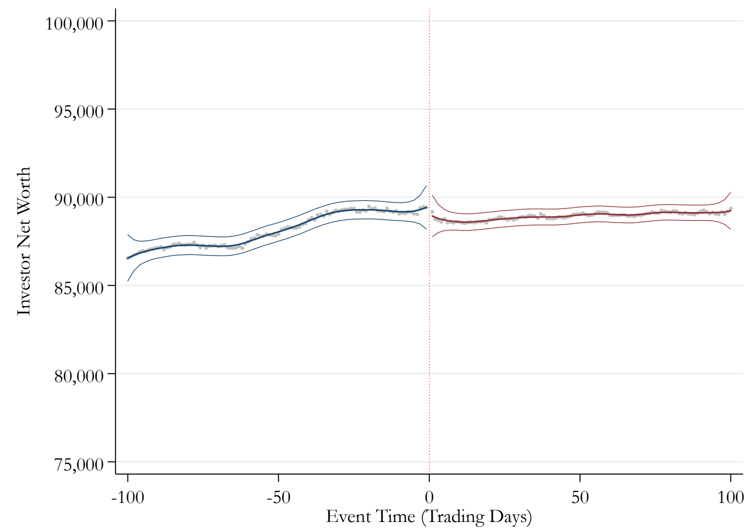
Institutional ownership and sales volatility

	Before Split			After Split			Difference		
	Obs	Mean	Std Dev	Obs	Mean	Std Dev	Obs	Mean	Std Dev
Institional Ownership	4,531	0.473	0.290	4,610	0.463	0.279	9,141	0.009	0.006
Sales Volatility	4,484	0.201	1.566	4,691	0.209	1.939	9,175	-0.008	0.037

Changes in institutional ownership and sales volatility after splits are economically small and insignificant

Change in investor base

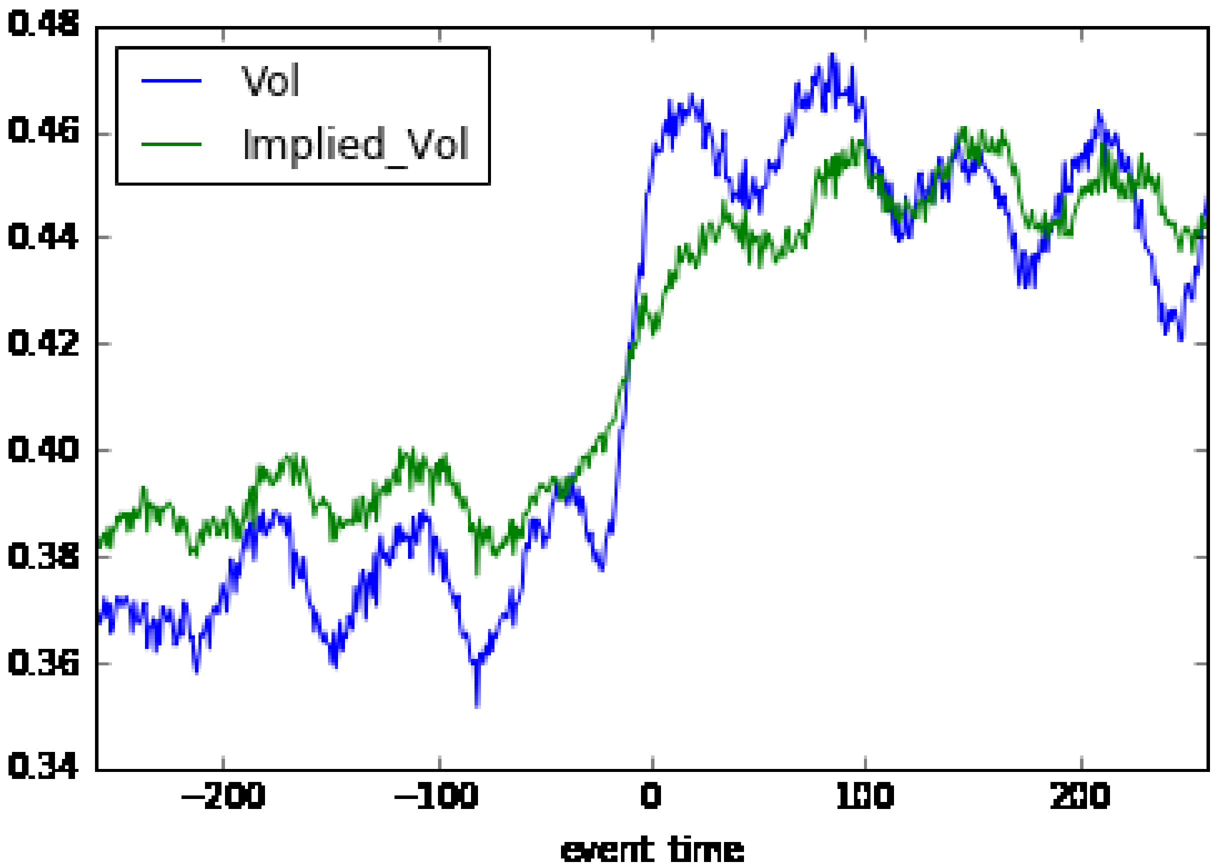




Splits are not associated with ***discontinuous*** changes in retail investor characteristics e.g. income and net worth

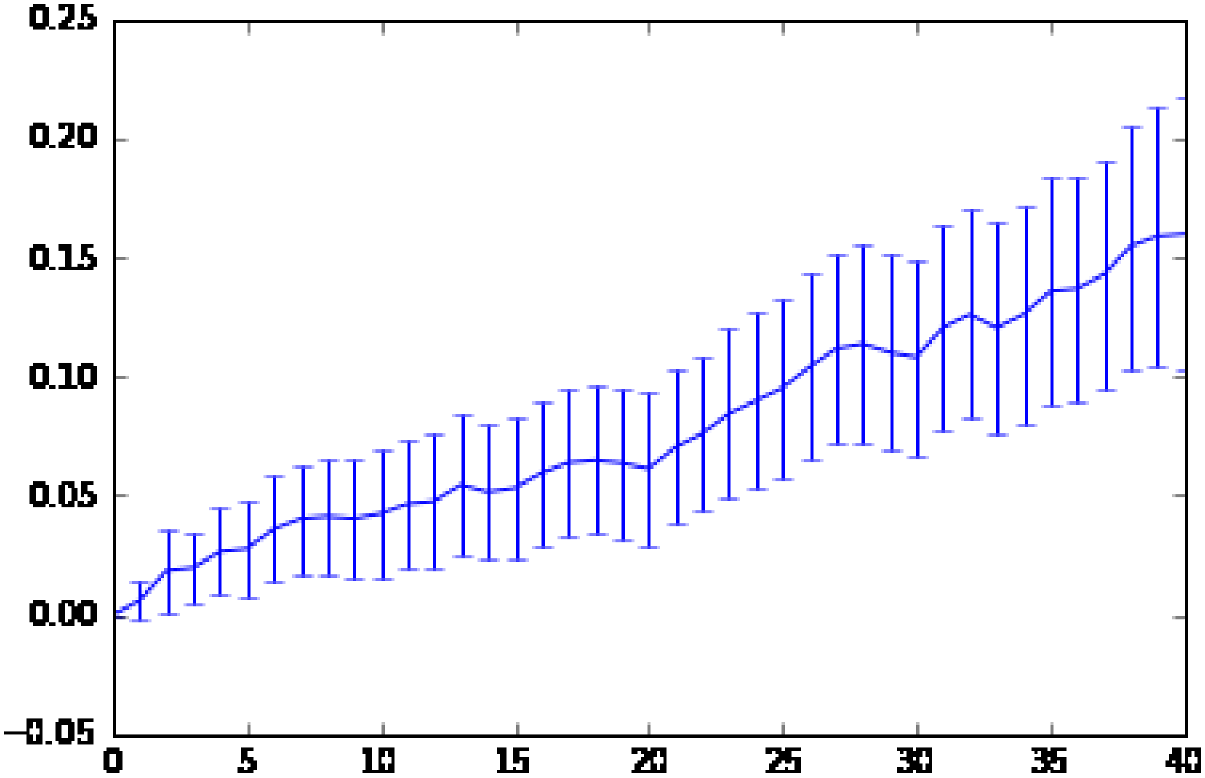
(data from Barber and Odean, 2000)

Option implied vol and actual vol



- Usually, we expect Implied Vol > Vol in option markets
- Results are consistent with option traders under-estimating the increase in volatility after splits

Trading strategy: buy straddles on split date



- Average 15 percent return within 40 days after split (does not account for transaction costs)

Extension: reactions to earnings surprises

Non-proportional thinking may distort reactions to news if the news itself is reported in nominal rather than the appropriate scaled units

Suppose a firm's EPS beats expectations by 5 cents per share

- This is a bigger positive surprise if the firm's price is \$20/share than if the firm's price is \$30/share
- Therefore, most academic papers measure earnings news as 5 cents scaled by lagged share price

However, the media usually reports the nominal EPS surprise…

CNBC earnings announcement news

Nominal vs. scaled earnings surprise

$$CAR^i_{[t-1,t+X]} = \beta_1 scaled\ sur^i_t + \beta_2 nominal\ sur^i_t + \cdots$$

- $nominal\ sur^i_t$ = actual earnings – forecasted earnings
- $scaled\ sur^i_t = nominal\ sur^i_t / price^i_{t-3}$
- Measure both as percentiles to make coefficients comparable

Consistent with a non-proportional thinking bias, we find that ***nominal surprise*** **predicts short run returns but only** ***scaled surprise*** **predicts long run return reactions**

Short run reactions to earnings surprises

	Cummulative Abnormal Return [-1,1]			
	(1)	(2)	(3)	(4)
	All	All	Large Cap	Small Cap
Surprise Scaled	18.44***			
	(2.879)			
Surprise Nominal	8.134***			
	(0.224)			
Percentile Surprise Scaled		0.0399***	0.0282***	0.0187***
		(0.00227)	(0.00358)	(0.00323)
Percentile Surprise Nominal		0.0304***	0.0283***	0.0613***
		(0.00216)	(0.00290)	(0.00343)
R-squared	0.033	0.070	0.058	0.077
Observations	217,731	217,731	91,125	126,606

In the short run, investors react as much or more to the nominal earnings surprise

Long run reactions to earnings surprises

	Cummulative Abnormal Return			
	(1)	(2)	(3)	(4)
	[-1,1]	[-1,25]	[-1,50]	[-1,75]
Percentile Surprise Scaled	0.0399***	0.0722***	0.0959***	0.109***
	(0.00227)	(0.00537)	(0.00739)	(0.00841)
Percentile Surprise Nominal	0.0304***	0.0142***	0.00252	-0.00451
	(0.00216)	(0.00503)	(0.00698)	(0.00798)
R-squared	0.070	0.029	0.021	0.016
Observations	217,731	217,731	217,731	217,731

In the long run, only the scaled earnings surprise affects prices

Over and underreaction by type of news

- Holding perception of the magnitude of the news constant, NPT predicts that investors **overreact** to the news in return units for lower-priced firms

  → Negative relation between volatility and price

  Applies to most types of news (e.g. macro news, sick CEO)

- If news is reported in nominal units that tend to be small for lower priced firms, NPT predicts that investors will perceive the magnitude of the news as less than the true magnitude, and **underreact** to the real news

  → Positive (or less negative) relation between volatility and price

Earnings vs. Non-Earnings Periods

	Log(Total Volatility)	Log(Idiosyncratic Volatility)		
	(1)	(2)	(3)	(4)
	All	All	Small Cap	Large Cap
Log(Lagged Price)	-0.218***	-0.240***	-0.299***	-0.181***
	(0.00954)	(0.00912)	(0.00701)	(0.0118)
Log(Lagged Price) × Earnings Announcement	0.0370***	0.0579***	0.0794***	0.0415***
	(0.00692)	(0.00698)	(0.00828)	(0.0133)
Size Category FE	Yes	Yes	Yes	Yes
Year-Quarter FE	Yes	Yes	Yes	Yes
R-squared	0.237	0.252	0.181	0.183
Observations	330,736	330,736	159,831	170,905

Consistent with NPT, the negative relation between volatility and price significantly reverses during earnings announcement windows [-1, +1]

- This pattern is not consistent with an alternative explanation in which speculative investors own low priced stocks and always introduce more volatility and overreact to all news

Contrast with the proportional-thinking bias

E.g. Pratt et al. 1979, Thaler 1980, Tversky and Kahneman 1981, Azar 2011, Bushong et al. 2017

Consumers are often willing to drive to another store to save $10 off a $20 calculator but not to save $10 off a $100 jacket

Consumers should think in levels, but partially focus on the discount as a proportion of the purchase price

Investors in financial markets should think proportionally, but partially think in levels and fail to scale by price

Conclusion

Non-proportional thinking: Investors think that news should correspond to a dollar change in price rather than a percentage change in price

- Return overreaction for low-priced stocks and underreaction for high priced stocks

Economic magnitudes are large

- Non-proportional thinking can explain a significant portion of the "leverage effect" puzzle as well as the volatility-size and beta-size relations in the data

Non-proportional thinking also distorts reactions to news that is itself reported in nominal rather than real units

Offers insight into the determinants of volatility, drift, and reversals

Contrast Effects

- Contrast effects: Value of previously-observed signal inversely biases perception of the next signal
- Abundant experimental evidence in psychology
  - Crimes viewed as less serious after exposure to more egregious crimes (Pepitone and DiNubile 1976)
  - Men rate female students as less attractive if the men recently viewed pictures of very beautiful actresses (Kenrick and Gutierres 1980)
- Contrast effects in popular culture
  - “A tough act to follow” / “Pale in comparison”
  - Literary foils
  - “Ugly friend” makes you look hotter

Contrast Effects

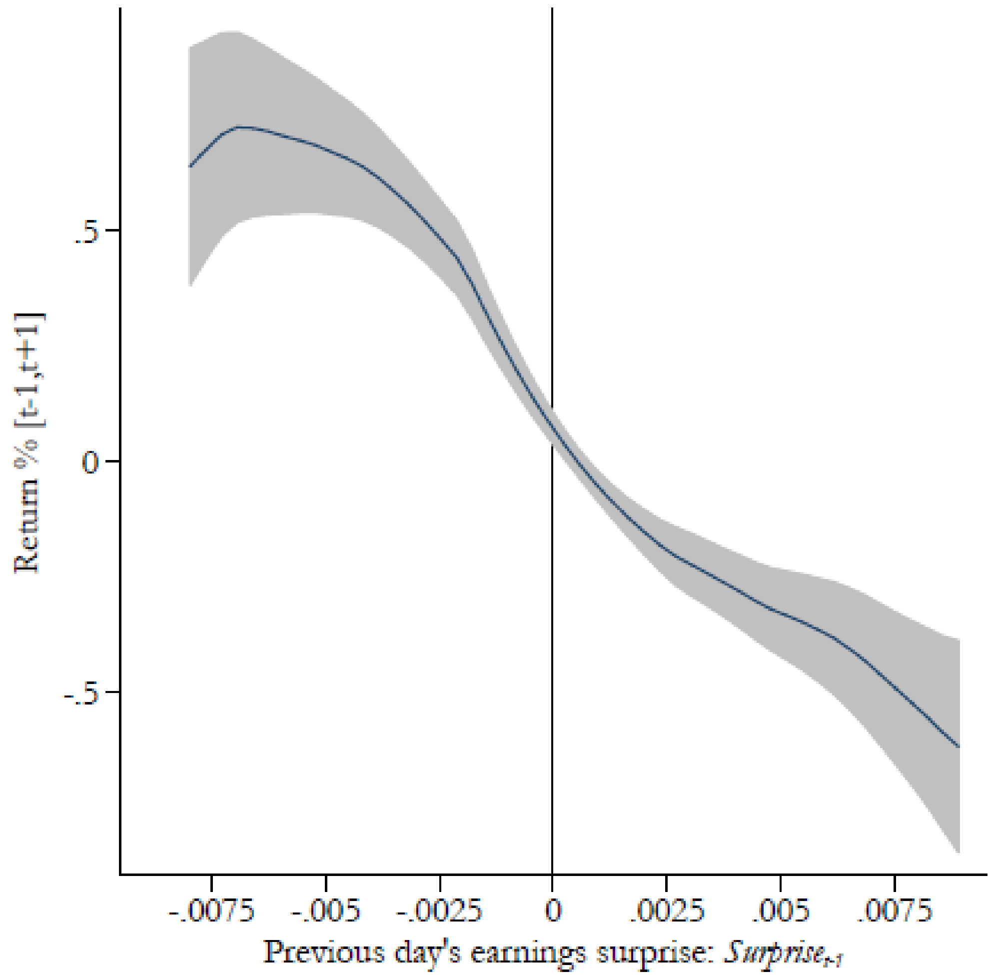
Contrast Effects

Contrast Effects

- Hartzmark and Shue (2016) examine the impact of contrast effects on the stock return reaction to firm earnings announcements

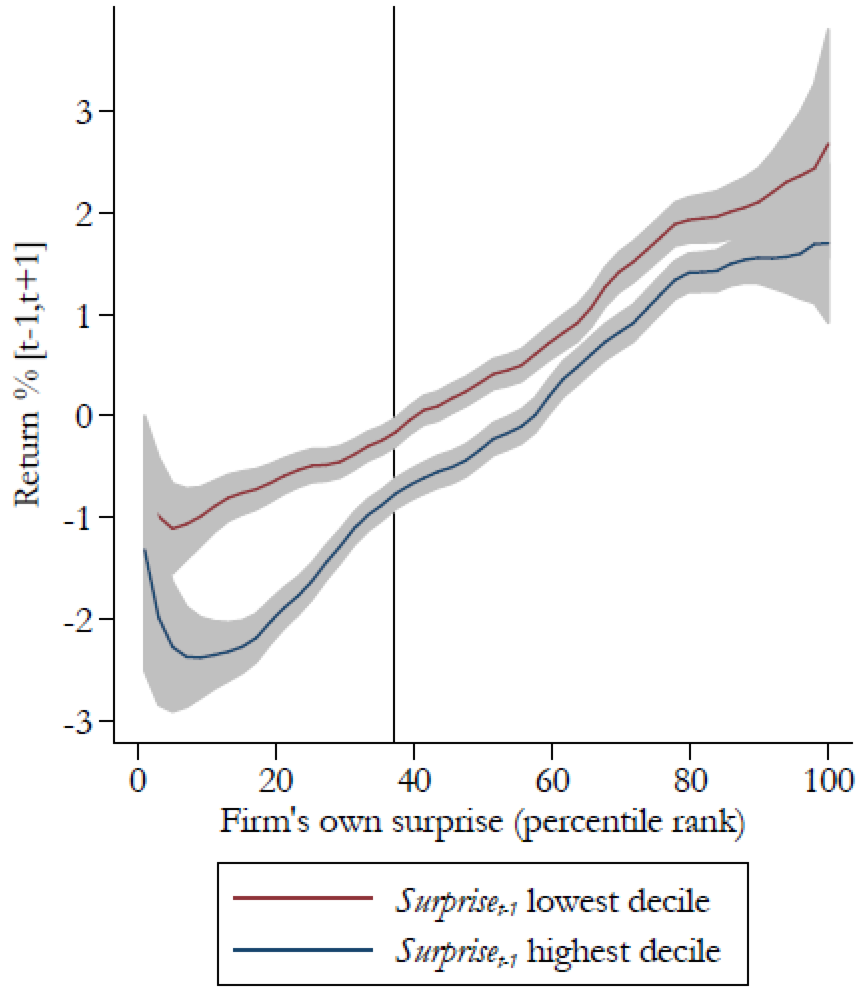
- Contrast effects => Negative relation between yesterday's earnings surprise and the return reaction to today's earnings news, holding today’s earnings news constant
    - A high surprise yesterday makes any surprise today look slightly worse than the same surprise today would appear if yesterday's surprise had been lower

Contrast Effects



Source: Hartzmark and Shue 2016

Contrast Effects



Source: Hartzmark and Shue 2016

Contrast Effects: Long Run Reversal

	$[t-1, t+10]$	$[t-1, t+20]$	$[t-1, t+30]$	$[t-1, t+40]$	$[t-1, t+50]$
	(1)	(2)	(3)	(4)	(5)
$Surprise_{t-1}$	-0.837**	-0.831**	-0.317	-0.0945	0.493
	(0.405)	(0.409)	(0.497)	(0.561)	(0.686)
Own $surprise_{it}$ controls	Yes	Yes	Yes	Yes	Yes
Year-month FE	Yes	Yes	Yes	Yes	Yes
R^2	0.0616	0.0465	0.0375	0.0373	0.0359
Observations	75736	75567	75362	74995	74149

	$[t+1, t+10]$	$[t+1, t+20]$	$[t+1, t+30]$	$[t+1, t+40]$	$[t+1, t+50]$
	(1)	(2)	(3)	(4)	(5)
$Surprise_{t-1}$	0.00969	0.0371	0.472	0.755	1.327*
	(0.340)	(0.371)	(0.482)	(0.559)	(0.677)
Own $surprise_{it}$ controls	Yes	Yes	Yes	Yes	Yes
Year-month FE	Yes	Yes	Yes	Yes	Yes
R^2	0.0228	0.0215	0.0215	0.0247	0.0272
Observations	75783	75607	75397	75028	74179

- Mispricing corrects within 25 to 50 trading days

Source: Hartzmark and Shue 2016

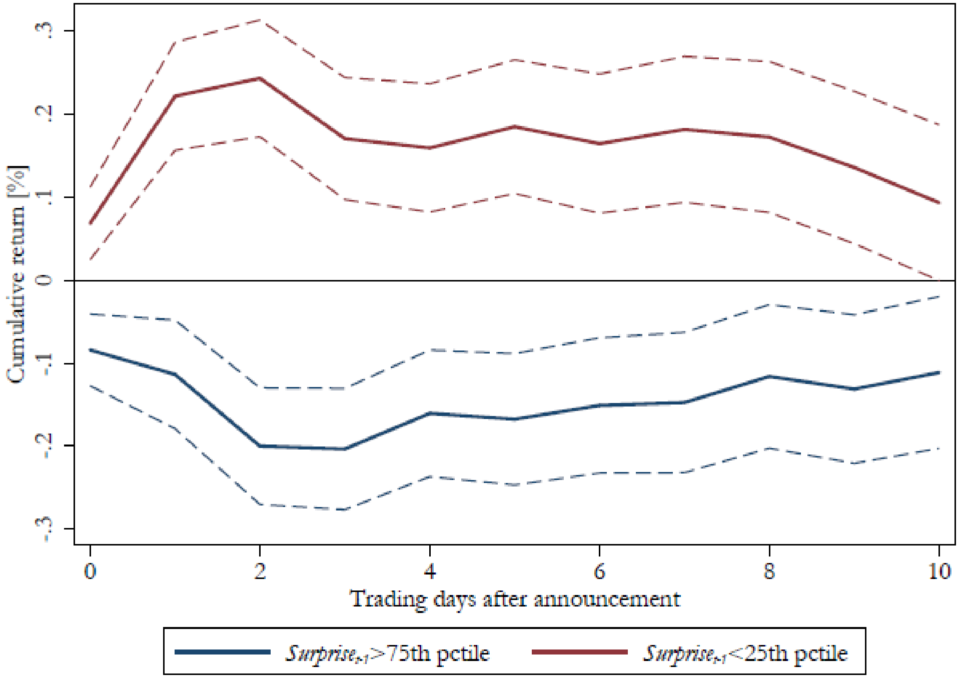
Contrast Effects

	$Surprise_{it}$		Open-to-open ret $[t-1]$	
	(1)	(2)	(3)	(4)
$Surprise_{t-1}$	0.157***	0.0115	0.128	0.0655
	(0.0603)	(0.0602)	(0.155)	(0.145)
Own $surprise_{it}$ controls	No	No	No	No
Year-month FE	No	Yes	No	Yes
R^2	0.00204	0.0324	0.000153	0.0253
Observations	75923	75923	61732	61732

- The mispricing occurs even though yesterday's earnings news does not predict today's earnings news, after accounting for earnings season averages.

Source: Hartzmark and Shue 2016

Contrast Effects: Trading Strategy



- Bad earnings news today: Long firms announcing tomorrow
- Good earnings news today: Short firms announcing tomorrow
- Trading only large caps: 20 basis points per day

Source: Hartzmark and Shue 2016

Related literature

Thinking about value in the wrong units

- Birru and Wang (2015, 2016): Investors mistakenly believe that low-priced stocks have more room to grow
- Shue and Townsend (2017): Thinking about the number rather than value of options contributed to the rise in CEO pay
- Baker and Wurgler (2004a,b), Baker et al. (2007), Hartzmark and Solomon (2017, 2018): Investors think about dividends and capital gains separately instead of total returns
- Money illusion with respect to nominal and real value of money: Fisher (1928); Benartzi and Thaler (1995)

We offer a new explanation of some known facts/puzzles regarding volatility, e.g. Dubofsky (1991), Dhar et al. (2004)

A simple model of non-proportional thinking

- P is current share price of a stock
- P_0 is the reference price in the minds of investors
- News is released, rational return reaction should be fraction δ
- Non-proportional thinking leads investors to think that news should move prices by a nominal amount X, such that X equals the rational return reaction if the stock's price equaled P_0, i.e. $\mathrm{X} = \delta P_0$
- $\theta \in [0,1]$ is extent of non-proportional thinking

Return reaction to news:

$$r = \theta \left(\frac{\delta P_0}{P} \right) + (1 - \theta)\delta$$

Model predictions

$$r = \theta\left(\frac{\delta P_0}{P}\right) + (1-\theta)\delta$$

- Overreaction to news for low-priced stocks ($P < P_0$), and eventual reversal as prices reflect fundamentals
- Underreaction to news for high-priced stocks and eventual drift in the direction of news as prices reflect fundamentals

- Higher total and idiosyncratic volatility for lower priced stocks
- Higher absolute beta for lower priced stocks (due to overreaction to market news)

Heterogeneity by institutional ownership

	(1) Log(Total Volatility)	(2) Log(Idiosyncratic Volatility)	(3) Log(|Beta|)
Log(Lagged Price)	-0.384***	-0.381***	-0.394***
	(0.00509)	(0.00511)	(0.00579)
Log(Lagged Price) × Lagged Inst. Ownership	0.169***	0.137***	0.167***
	(0.0108)	(0.0107)	(0.0124)
Lagged Inst. Ownership	-0.311***	-0.280***	-0.128***
	(0.0323)	(0.0314)	(0.0388)
Month FE	Yes	Yes	Yes
Size Category FE	Yes	Yes	Yes
R-squared	0.432	0.461	0.123
Observations	2,113,118	2,113,118	2,113,118

Volatility-price relation isn't driven by a correlation between price and institutional ownership

- But the magnitude of volatility-price relation declines with institutional ownership, consistent with limits to arbitrage

Upside and downside volatility

	Log(Mean Squared Returns)			Log(|Beta|)		
	(1)	(2)	(3)	(4)	(5)	(6)
	All	$\text{Ret}_i > 0$	$\text{Ret}_i < 0$	All	$\text{Ret}_{mkt} > 0$	$\text{Ret}_{mkt} < 0$
Log(Lagged Price)	-0.337***	-0.397***	-0.365***	-0.305***	-0.333***	-0.328***
	(0.00401)	(0.00374)	(0.00372)	(0.00538)	(0.00459)	(0.00471)
Month FE	Yes	Yes	Yes	Yes	Yes	Yes
Size Category FE	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.444	0.521	0.543	0.085	0.124	0.173
Observations	3,254,302	3,209,814	3,220,782	3,254,302	3,221,095	3,217,915

- Non-proportional thinking predicts that investors will overreact to both positive and negative news for lower priced stocks
- Lower share price is associated with greater upside and downside volatility, as well as greater upside and downside beta

Similar results with additional control variables

	Log(Total Volatility)					
	(1)	(2)	(3)	(4)	(5)	(6)
Log(Lagged Price)	-0.326***	-0.344***	-0.308***	-0.305***	-0.282***	-0.288***
	(0.00339)	(0.00441)	(0.00533)	(0.00539)	(0.00533)	(0.00544)
Log(Lagged Sales Volatility)			0.0574***	0.0379***	0.0260***	0.0268***
			(0.00191)	(0.00151)	(0.00133)	(0.00139)
Log(Lagged Market-to-Book)				0.192***	0.151***	0.145***
				(0.00744)	(0.00652)	(0.00671)
Log(Lagged Volume Turnover)					0.110***	0.109***
					(0.00228)	(0.00235)
Log(Lagged Leverage)						-0.00521***
						(0.00146)
Log(Lagged Market Cap)	No	Yes	Yes	Yes	Yes	Yes
ME Breakpoint FE	No	Yes	Yes	Yes	Yes	Yes
Log(Lagged Market Cap) × ME Breakpoint FE	No	Yes	Yes	Yes	Yes	Yes
Month FE	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.442	0.446	0.456	0.473	0.503	0.512
Observations	3,254,548	3,254,280	1,944,220	1,777,923	1,750,786	1,457,177

Adjusting for tick-size distortions

Subtract half a tick if price is up, and add half a tick if price is down

- These artificial prices round to actual prices but compress returns
  → Lower bound for true volatility absent tick-size constraints
- Difference between true volatility and lower bound is largest for low priced stocks

If tick-size effects drive our results, the price-volatility relation should disappear using this alternative volatility measure

	Log(Total Tick-Size Adjusted Volatility)			
	(1)	(2)	(3)	(4)
Log(Lagged Price)	-0.268***		-0.266***	-0.275***
	(0.00348)		(0.00460)	(0.00434)
Log(Lagged Size)		-0.122***	-0.00157	
		(0.00231)	(0.00325)	
Month FE	Yes	Yes	Yes	Yes
Size Category FE	No	No	No	Yes
R-squared	0.358	0.285	0.358	0.361
Observations	3,254,302	3,254,302	3,254,302	3,254,302

Summary statistics

	Obs	Mean	Median	Std Dev
Total Volatility (Annualized)	3,254,302	0.510	0.390	0.450
Idiosyncratic Volatility (Annualized)	3,254,302	0.434	0.323	0.404
Market Beta	3,254,302	0.977	0.827	3.540
Price	3,254,302	18.85	13.50	19.13
Market Capitalization (Millions)	3,254,302	1179.9	64.47	8800.6
Insitutional Ownership	2,165,251	0.346	0.272	0.292
Sales Volatility	2,316,178	0.273	0.0966	0.691
Volume Turnover	2,996,292	0.0882	0.0382	0.204
Market-to-Book Ratio	2,240,583	1.977	1.252	18.19
Book Leverage	2,130,604	0.232	0.191	0.278

Leverage does not explain the "leverage effect"

- Leverage effect: firms with low price or negative returns have high vol
  - Potential explanation: As asset value declines, equity becomes more levered, so equity volatility and beta increases
- But, we find similar patterns for firms with zero leverage

	(1) Log(Total Volatility)	(2) Log(Idiosyncratic Volatility)	(3) Log(|Beta|)
Log(Lagged Price)	-0.286***	-0.290***	-0.298***
	(0.00775)	(0.00774)	(0.00856)
Month FE	Yes	Yes	Yes
Size Category FE	Yes	Yes	Yes
R-squared	0.337	0.363	0.110
Observations	224,571	224,571	224,571

Within-firm changes in price

	Log(Total Volatility)			
	(1)	(2)	(3)	(4)
Log(Lagged Price)	-0.260***		-0.261***	-0.274***
	(0.00395)		(0.00477)	(0.00403)
Log(Lagged Size)		-0.160***	0.000476	
		(0.00334)	(0.00383)	
Stock FE	Yes	Yes	Yes	Yes
Month FE	Yes	Yes	Yes	Yes
Size Category FE	No	No	No	Yes
R-squared	0.588	0.565	0.588	0.588
Observations	3,254,302	3,254,302	3,254,302	3,254,302

Controlling for firm FE (identifying off of within-firm changes in price)

	Log(Total Volatility)					
	(1) 2-Month	(2) 4-Month	(3) 6-Month	(4) 8-Month	(5) 10-Month	(6) 12-Month
Lagged Return	-0.114***	-0.0952***	-0.0835***	-0.0745***	-0.0647***	-0.0557***
	(0.0253)	(0.0204)	(0.0158)	(0.0126)	(0.0106)	(0.00899)
Month FE	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.184	0.185	0.185	0.185	0.186	0.186
Observations	2,966,196	2,966,196	2,966,196	2,966,196	2,966,196	2,966,196

Recent returns have a stronger negative relation with volatility

Splits (idiosyncratic volatility)